Mini-Matlab Lesson 4: Numerical integration

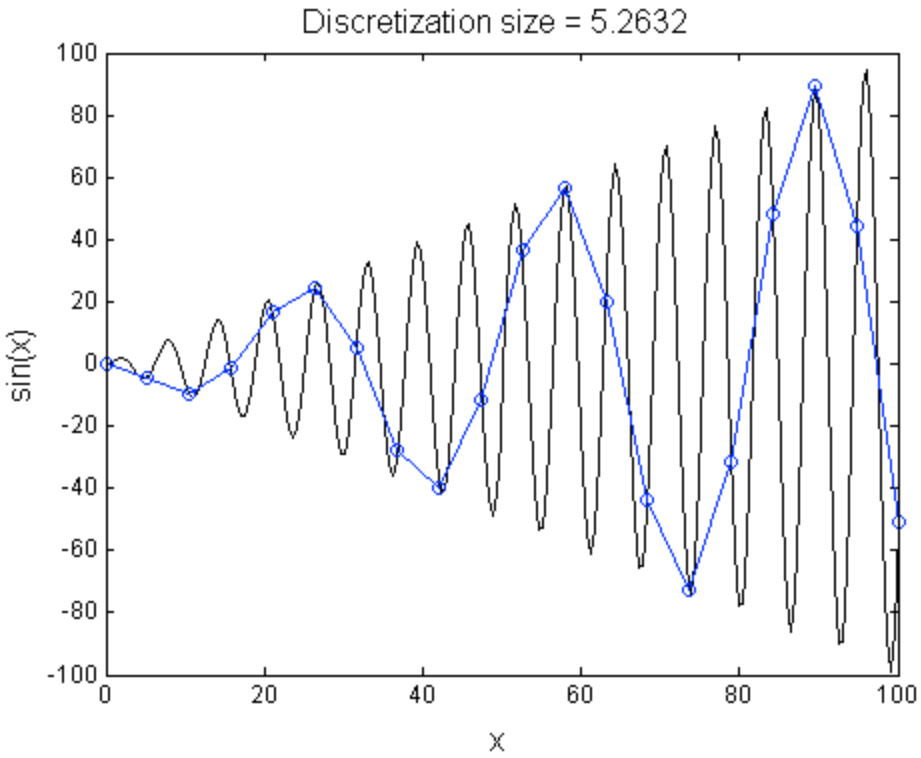
How do you take the (definite) integral of a function?

Contents

Setup

We begin with our example from last lesson

```
clear
close all

myfunc = @(x) x.*sin(x);
myfuncint = @(x) sin(x) - x.*cos(x);

N = 20;

x = linspace(0, 100, N);    % Rough mesh
f = myfunc(x);

xs = linspace(0, 100, 200); % Fine mesh for comparison

figure(1)
plot(xs, myfunc(xs), 'k');   % 'k' creates a black line
hold on                      % Hold the plot
plot(x, f, 'bo-');           % 'bo-' creates a blue line with circles
hold off
xlabel('x', 'FontSize', 16);
ylabel('sin(x)', 'FontSize', 16);
title(['Discretization size = ', num2str(x(2)-x(1))], 'FontSize', 16);
```



Trapezoid rule

The idea of this method is to partition the interval [a, b] into N subintervals separated by [x1, x2, x3, ..., xN]. On each interval, we estimate the area using a trapezoid. This is given by

$$\int_{x_k}^{x_{k+1}} f(x)\,dx = \frac{x_{k+1}-x_k}{2}[f(x_k)+f(x_{k+1})]$$

We can thus write the entire integral as

$$I = \sum_{k=1}^{N-1} \frac{x_{k+1}-x_k}{2}[f(x_k)+f(x_{k+1})]$$

If we assume that the discretization intervals are equal, then

$$I = h\sum_{k=1}^{N-1}[f(x_k)+f(x_{k+1})] = \frac{h}{2}[f(a)+f(b)] + h\sum_{k=1}^{N-1} f(x_k)$$

For the remainder, I'll assume the intervals are unequal (the speed-up will be minimal)

```
I = sum((f(1:end-1)+f(2:end)).*(x(2:end)-x(1:end-1))/2);

disp(['Numerical value = ', num2str(I)]);
disp(['Exact value = ', num2str(myfuncint(x(end))-myfunc(x(1)))]);
```

```
Numerical value = 390.9629
Exact value = -86.7383
```

Two notes...

It can be proven that for an equally spaced grid, the error is

$$\text{error} = -\frac{(b-a)^3}{12N^2} f''(\xi)$$

where $\xi \in [a, b]$.

Matlab also has a built-in trapezoidal integration. It will be a bit slower than if you code it by hand because it has all sorts of error catching rules

```
I1 = trapz(x, f)
I2 = sum((f(1:end-1)+f(2:end)).*(x(2:end)-x(1:end-1))/2)
```

```
I1 =

  390.9629


I2 =

  390.9629
```

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