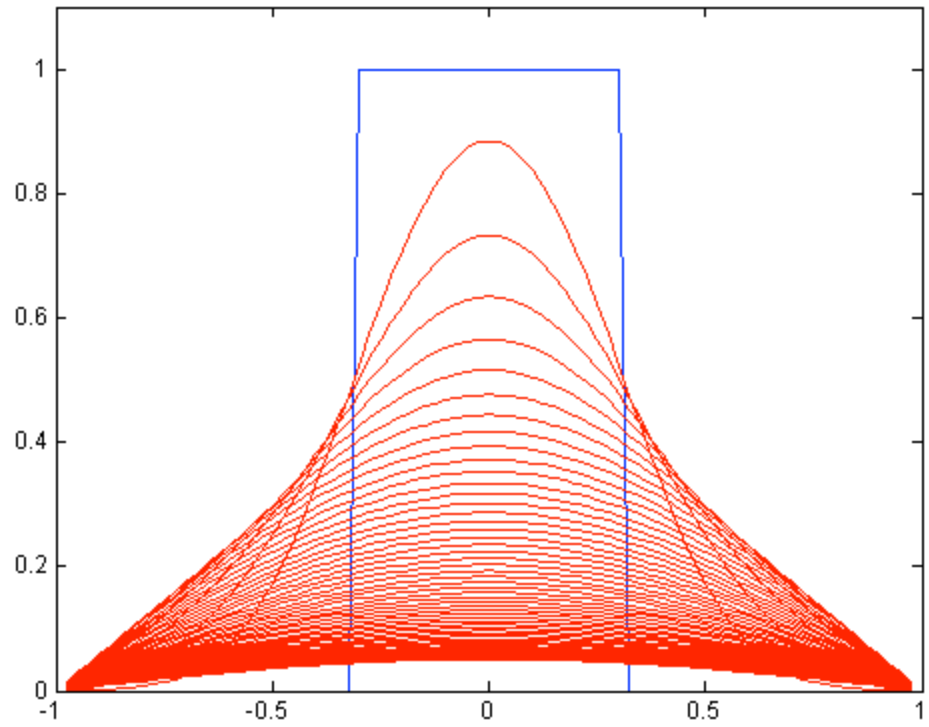
Explicit solution of heat equation

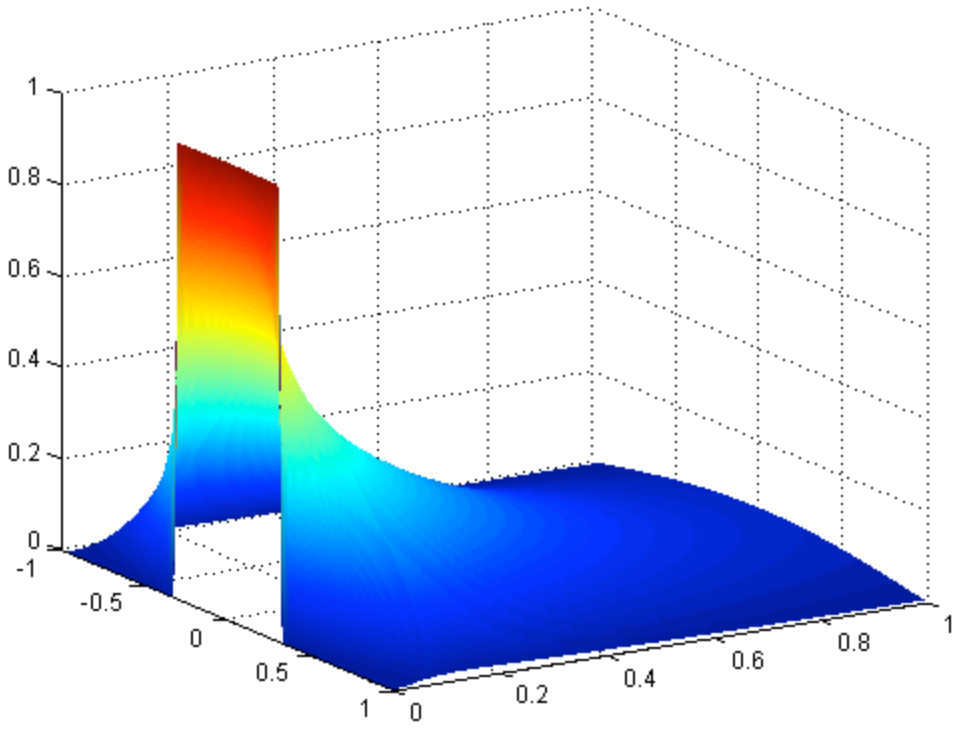
$u_t = u_{xx},$

with

$u(-1) = u(1) = 0.$

```
  clear; close all;

% Grid and initial data:
  h = .025;                     % space step
  k = .4*h^2;                   % time step (try .4 -> .51!)
  x = (-1+h:h:1-h)';
  t = [0:k:1];
  u = abs(x)<.3;                % initial data: square wave

% Matrix to execute finite difference operation:
  L = length(x);
  a = (1-2*k/h^2);
  b = k/h^2;
  main = a*ones(L,1);
  off  = b*ones(L-1,1);
  A = diag(main) + diag(off,1) + diag(off,-1);
% A(1,end) = b; A(end,1) = b;  % comment this in for periodic BCs

% Time-stepping:
  U = zeros(length(t), length(x));
  U(1,:) = u;

  figure
  plot(x, u, 'b');
  hold on;
  for j = 2:length(t)
    u = A*u;

    U(j,:) = u;
    if mod(j,80) == 0
        plot(x, u, 'r');
    end
  end
  hold off;
  ylim([0 1.1]);

  figure
  s = surf(x, t, U);
  view([58 20]);
  set(s, 'EdgeColor', 'none', 'FaceColor', 'interp');
```





Published with MATLAB® 7.12

Explicit solution of 4th-order diffusion equation:

$u_t = -u_{xxxx}$,

with

$u(-1) = u(1) = u'(-1) = u'(1) = 0$.

```
% Grid, initial data, and plotting setup:
  h = .025;
  x = (-1+h:h:1-h)';
  u = abs(x)<.3;
  hold off, shg
  plt = plot(x,u,'linewidth',4);
  axis([-1 1 -.1 1.15]), grid
  k = input('k? (e.g. 1e-8, 4e-8, 5e-8, 4.88e-8, 4.89e-8) ');

% Matrix for finite differencing:
  L = length(x);
  a = (1-6*k/h^4);
  b = 4*k/h^4;
  c = -k/h^4;
  main = a*ones(L,1);
  off  = b*ones(L-1,1);
  off2 = c*ones(L-2,1);
  A = diag(main) + diag(off,1) + diag(off,-1) ...
                 + diag(off2,2) + diag(off2,-2);

% Time-stepping:
  t = 0;
  while 1
    u = A*u;
    t = t+k;
    set(plt,'ydata',u)
    title(['t = ' num2str(t)],'fontsize',20)
    drawnow
    pause;
  end
```

Published with MATLAB® 7.12

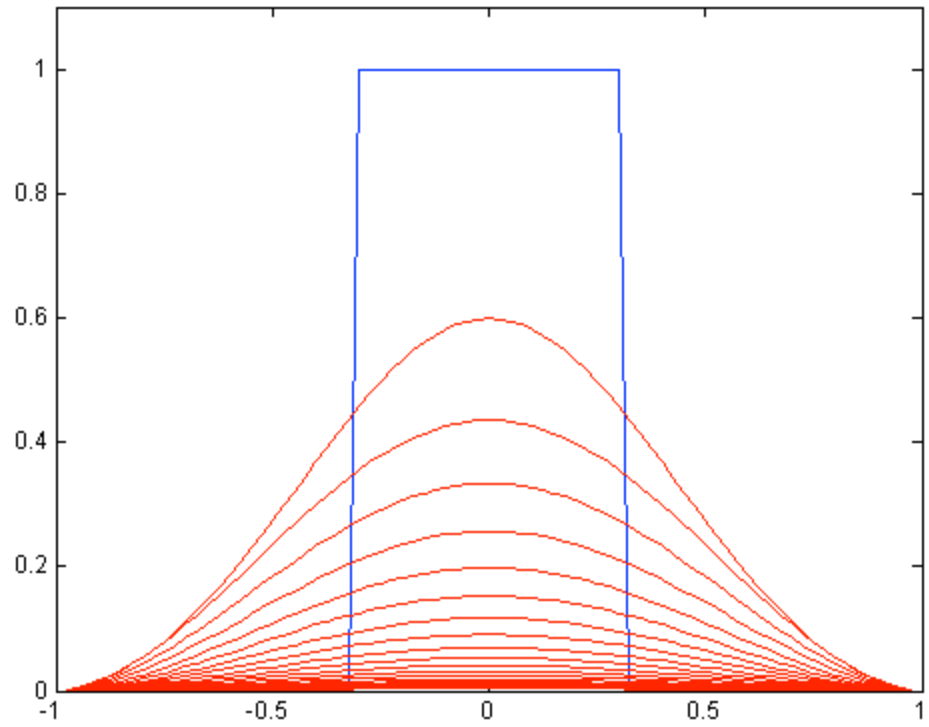
Implicit solution of 4th-order diffusion equation:

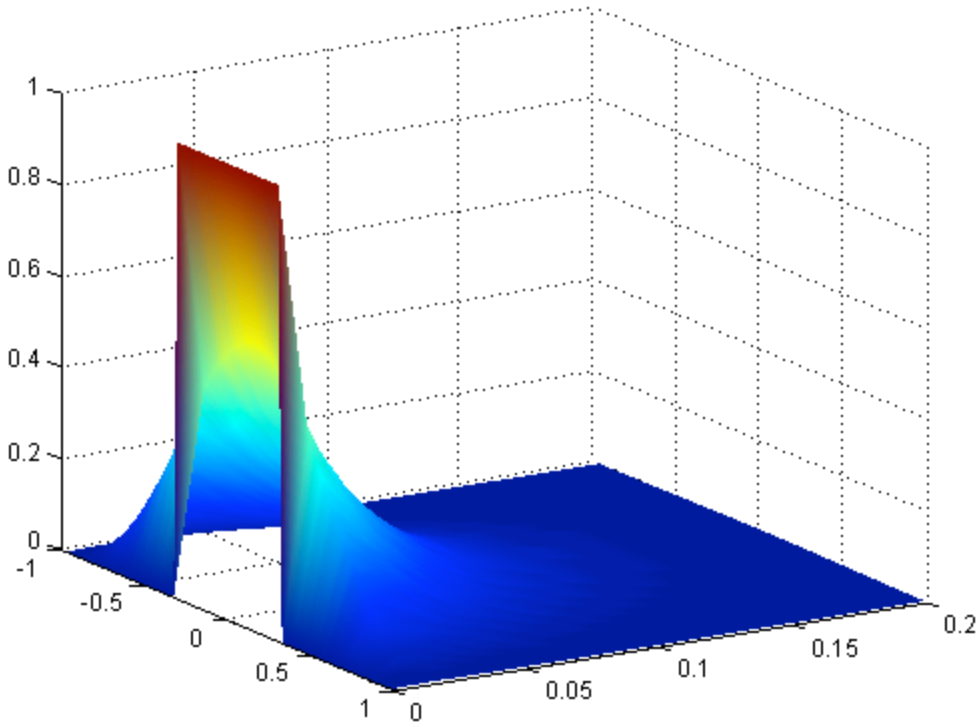
$u_t = -u_{xxxx}$,

with

$u(-1) = u(1) = u'(-1) = u'(1) = 0$.

```
clear; close all;

% Grid, initial data, and plotting setup:
  h = .025;
  x = (-1+h:h:1-h)';
  u = abs(x)<.3;
  k = 0.01;
  t = [0:k:0.2];

% Matrix for finite differencing:
  L = length(x);
  a = (1+6*k/h^4);        %  note that the signs have changed
  b = -4*k/h^4;
  c = k/h^4;
  main = a*ones(L,1);
  off  = b*ones(L-1,1);
  off2 = c*ones(L-2,1);
  B = diag(main) + diag(off,1) + diag(off,-1) ...
                 + diag(off2,2) + diag(off2,-2);

% Time-stepping:
  U = zeros(length(t), length(x));
  U(1,:) = u;

  figure
  plot(x, u, 'b');
  hold on;
  for j = 2:length(t)
    u = B\u;              %  note B\u, not A*u
    U(j,:) = u;
    plot(x, u, 'r');
  end
  hold off;
  ylim([0 1.1]);

  figure
  s = surf(x, t, U);
  view([58 20]);
  set(s, 'EdgeColor', 'none', 'FaceColor', 'interp');
```





Published with MATLAB® 7.12