

## Mini-Matlab Lesson 19: Separation of variables and Fourier series

We consider the heat distribution of a 1D bar beginning from nonsmooth initial conditions, with

$$u_t = Du_{xx}$$

$$u(0, t) = 0 = u(L, t)$$

$$u(x, 0) = f(x)$$

where

$$f(x) = -x \text{...for } 0 \leq x \leq 1/5$$

$$f(x) = x - 2/5 \text{...for } 1/5 < x \leq 7/10$$

$$f(x) = 1 - x \text{...for } 7/10 < x \leq 1$$

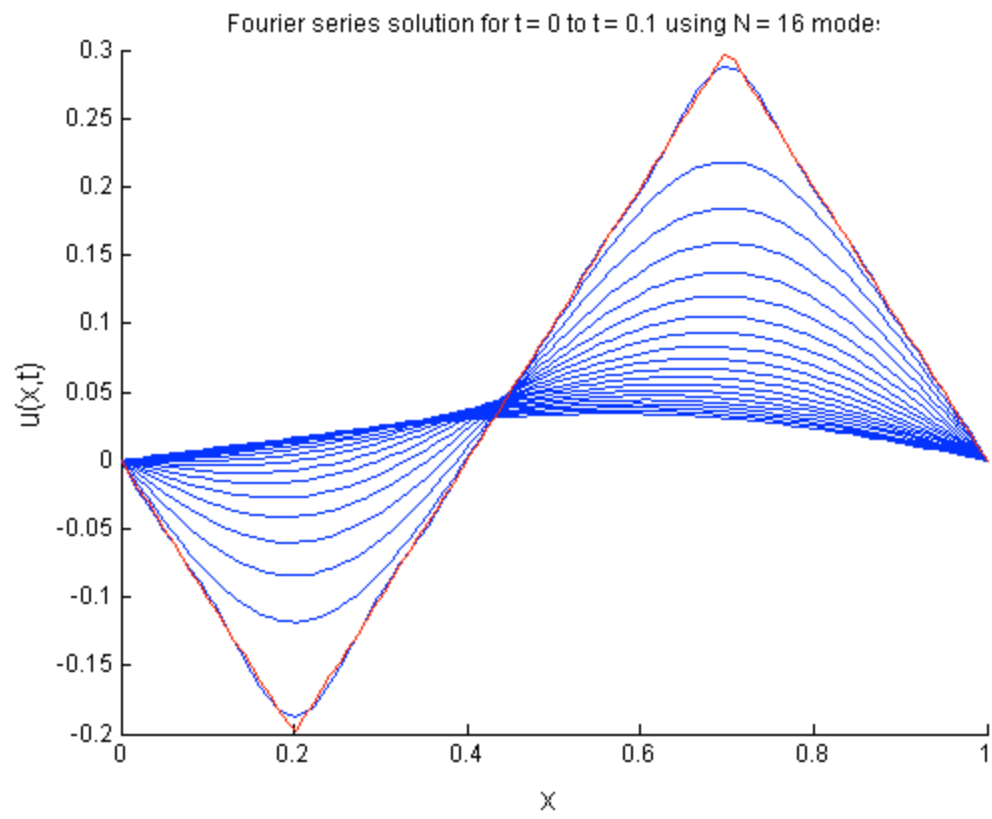
```
clear
close all

N = 16;
D = 1;
L = 1;

fwd = @(x)myfunc(x);
a = fouriersin(fwd, 1, N);

x = linspace(0, L, 100);
t = linspace(0, 0.1, 20);
[X,T] = meshgrid(x,t);
U = 0*X;

figure
hold on;
for j = 1:length(t)
    for k = 1:N
        lam = k*pi/L;
        U(j,:) = U(j,:) + a(k)*exp(-lam^2*D*t(j))*sin(lam*x);
    end
    plot(x,U(j,:), 'b');
end
plot(x, fwd(x), 'r');
hold off
xlabel('x', 'FontSize', 16);
ylabel('u(x,t)', 'FontSize', 16);
title('Fourier series solution for t = 0 to t = 0.1 using N = 16 modes');
```



```
% Function used in the heat equation initial condition of lectures
```

```
function f = myfunc(x)
```

```
f = 0*x;
```

```
ind = (x <= 1/5);
```

```
f(ind) = -x(ind);
```

```
ind = (x>1/5) & (x<= 7/10);
```

```
f(ind) = x(ind) - 2/5;
```

```
ind = x > 7/10;
```

```
f(ind) = 1-x(ind);
```

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```

% Compute the integral
%
%  $a_n = \frac{2}{L} \int_0^L f(x) \sin(n\pi x / L) dx$ 
%
% where f is a function
%
% The integration is performed using Matlab's 'quad' function which uses
% Simpson's method

function a = fouriersin(f, L, N)

    a = zeros(1, N);
    for k = 1:N
        g = @(x)f(x).*sin(k*pi*x/L);
        a(k) = 2/L*quad(g, 0, L);
    end
end

```

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### Mini-Matlab Lesson 19: Separation of variables and Fourier series 2

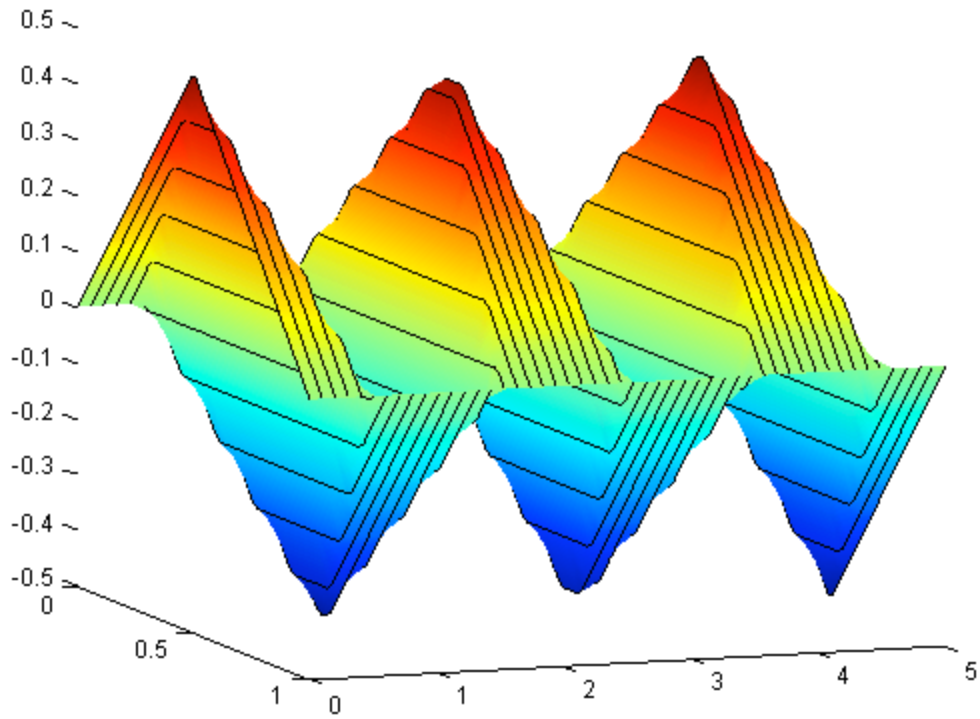
Fourier series solution for a plucked finite string

```
clear;
close all;

N = 32;
x = linspace(0, 1, 50);
t = linspace(0, 5, 50);
[X,T] = meshgrid(x,t);
U = 0*X;

f = figure;
hold on;
for j = 1:length(t)
    tmp = 0;
    for k = 0:N
        tmp = 4/pi^2*(-1)^k*cos((2*k+1)*pi*t(j))*sin((2*k+1)*pi*x)/(2*k+1)^2;
        U(j,:) = U(j,:) + tmp;
    end
    plot3(x, t(j)*ones(1,length(x)), U(j,:), 'k');
end

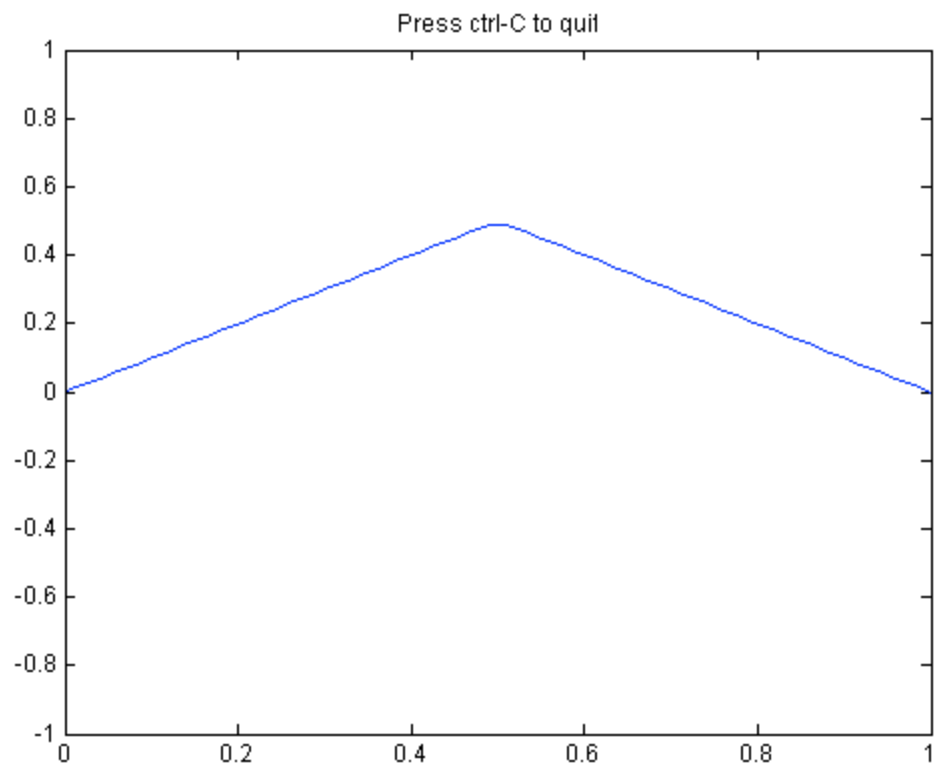
s = surf(X,T,U);
set(s, 'EdgeColor', 'none', 'FaceColor', 'interp');
view([70, 10]);
```



### Loop me

```
figure
j = 0;

while j <= 50
    plot(x,U(mod(j,length(t))+1,:), 'b');
    title('Press ctrl-C to quit');
    ylim([-1 1]);
    pause(0.1);
    drawnow;
    j = j + 1;
end
```



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