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### Mini-Matlab Lesson 18: Fourier series 1

Fourier approximation to  $f(x) = -1$  on  $[0, 1]$

```
clear;
close all;

N = 20;
x = linspace(0, 1, 200);
L = 1;

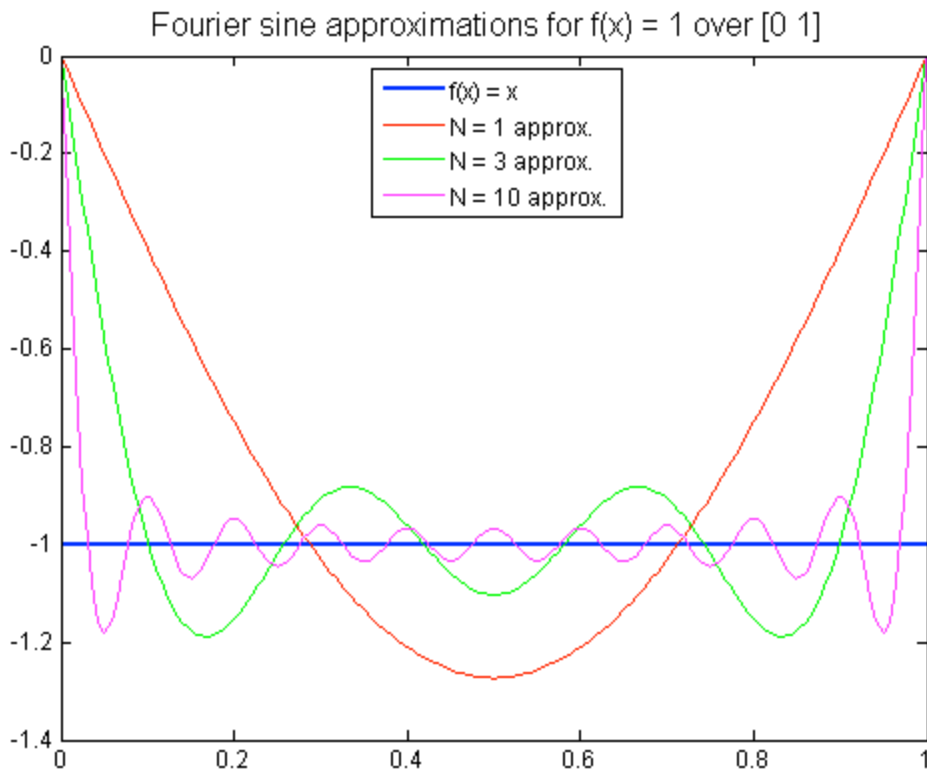
a = [];
f = zeros(1, length(x));

plot([0 1], [-1 -1], 'b', 'LineWidth', 2);
hold on;

for k = 1:N
    a(k) = -2*(1-(-1)^k)/(k*pi);
    f = f + a(k)*sin(pi*k*x/L);

    if k == 1
        plot(x, f, 'r');
    elseif k == 5
        plot(x, f, 'g');
    elseif k == 20
        plot(x, f, 'm');
    end
end

%ylim([-2 2]);
hold off;
title('Fourier sine approximations for f(x) = 1 over [0 1]', 'FontSize', 16);
legend('f(x) = x', 'N = 1 approx.', 'N = 3 approx.', 'N = 10 approx.', 'Location', '');
```



### Solution of the heat equation

We solve the heat equation for the one-dimensional bar with conditions  $u = 0$  at  $x = 0, 1$  and beginning with the initial distribution  $u = -1$ .

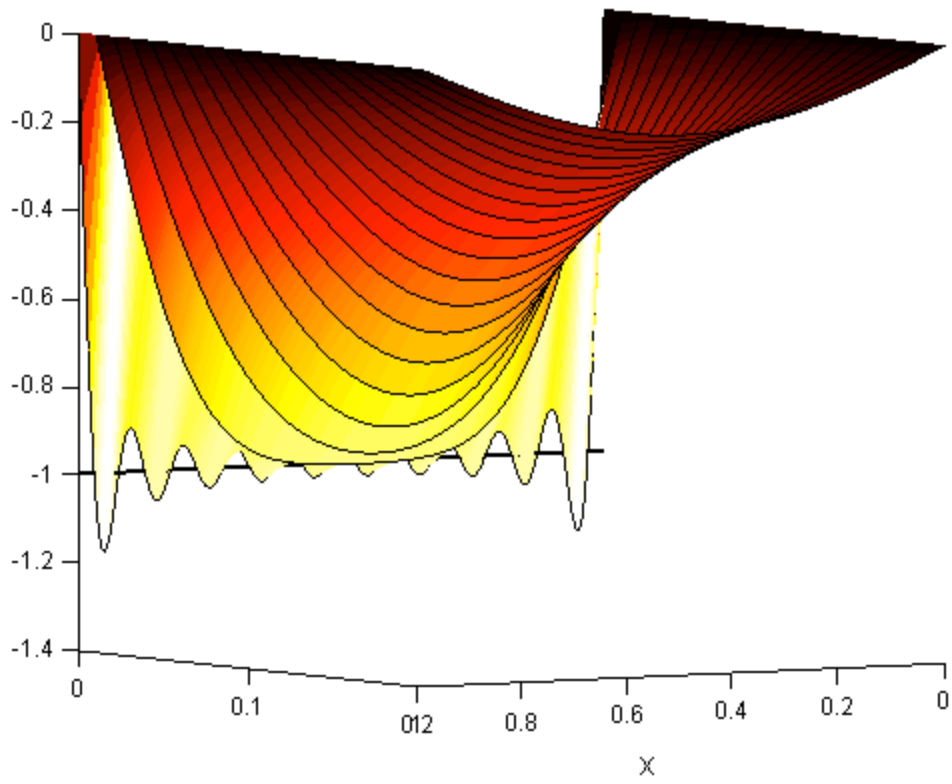
```
t = linspace(0, 0.2, 20);
[X, T] = meshgrid(x,t);

U = 0*X;

figure;
plot3([0 1], [0 0], [-1 -1], 'LineWidth', 2);
hold on;
for j = 1:length(t)
    for k = 1:N
        U(j,:) = U(j,:) + a(k)*exp(-(pi*k/L)^2*t(j))*sin(pi*k*x/L);
    end
    plot3(x, t(j)*ones(1,length(x)), U(j,:), 'k', 'LineWidth', 1);
end

S = surf(X,T,U, -U);
set(S, 'EdgeColor', 'none');
colormap(hot);
view([147 4]);
xlabel('x', 'FontSize', 16);
ylabel('y', 'FontSize', 16);
title('Solution of the heat equation for a bar', 'FontSize', 16);
```

Solution of the heat equation for a bar



## Mini-Matlab Lesson 18: Fourier series 2

We plot the Fourier approximations for

$$f(x) = x - x^3$$

```
clear;
close all;

N = 10;
x = linspace(-5, 5, 100);
L = 1;

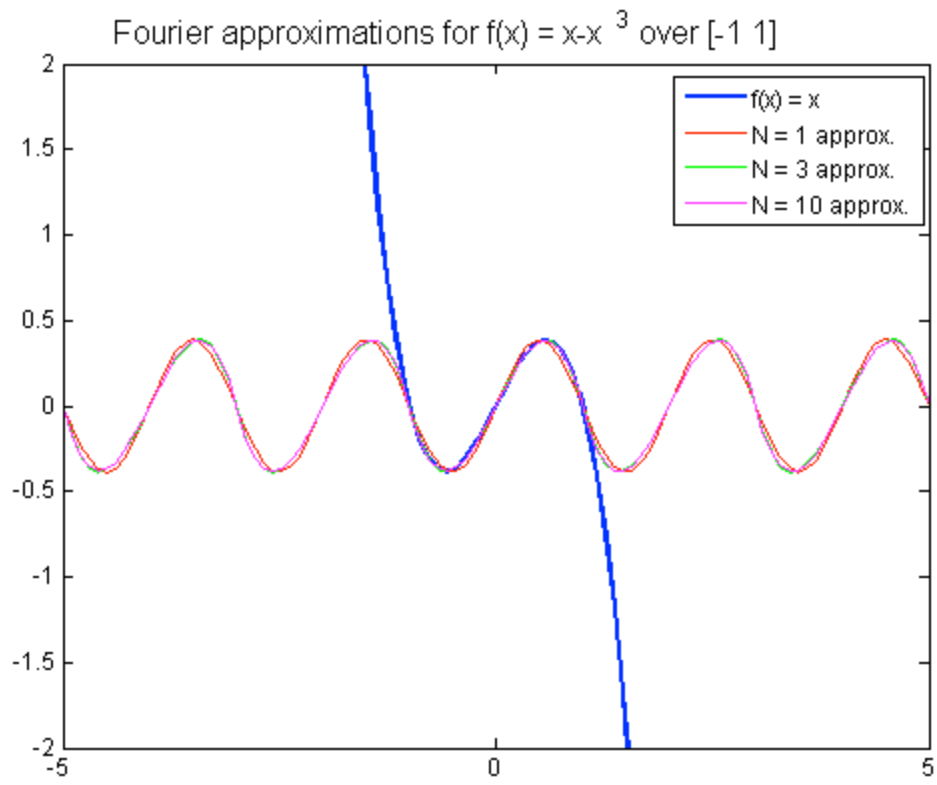
a = [];
f = zeros(1, length(x));

plot(x, x-x.^3, 'b', 'LineWidth', 2);
hold on;

for k = 1:N
    a(k) = -12*(-1)^k/(k*pi)^3;
    f = f + a(k)*sin(pi*k*x/L);

    if k == 1
        plot(x, f, 'r');
    elseif k == 3
        plot(x, f, 'g');
    elseif k == 10
        plot(x, f, 'm');
    end
end

hold off;
ylim([-2 2]);
hold off;
title('Fourier approximations for f(x) = x-x^3 over [-1 1]', 'FontSize', 16);
legend('f(x) = x', 'N = 1 approx.', 'N = 3 approx.', 'N = 10 approx.');
```



## Mini-Matlab Lesson 18: Fourier series 2

We plot the Fourier approximations for

$$f(x) = x$$

```
clear;
close all;

N = 10;
x = linspace(-5, 5, 800);
L = 1;

a = [];
f = zeros(1, length(x));

plot(x, x, 'b', 'LineWidth', 2);
hold on;

for k = 1:N
    a(k) = -2*(-1)^k/(k*pi);
    f = f + a(k)*sin(pi*k*x/L);

    if k == 1
        plot(x, f, 'r');
    elseif k == 3
        plot(x, f, 'g');
    elseif k == 10
        plot(x, f, 'm');
    end
end

ylim([-2 2]);
hold off;
title('Fourier approximations for f(x) = x-x^3 over [-1 1]', 'FontSize', 16);
legend('f(x) = x', 'N = 1 approx.', 'N = 3 approx.', 'N = 10 approx.');
```

