Mini-Matlab Lesson 16: Nonlinear transport equations (Q1)

We plot characteristics and solutions for the PDE

$u_t + uu_x = 0$

with $u(0,x) = f(x) = \pi/2 + tan^{-1}(x)$.

The solution is $u(t,x) = f(\xi) = f(x - ut)$

Contents

Plot of the characteristics

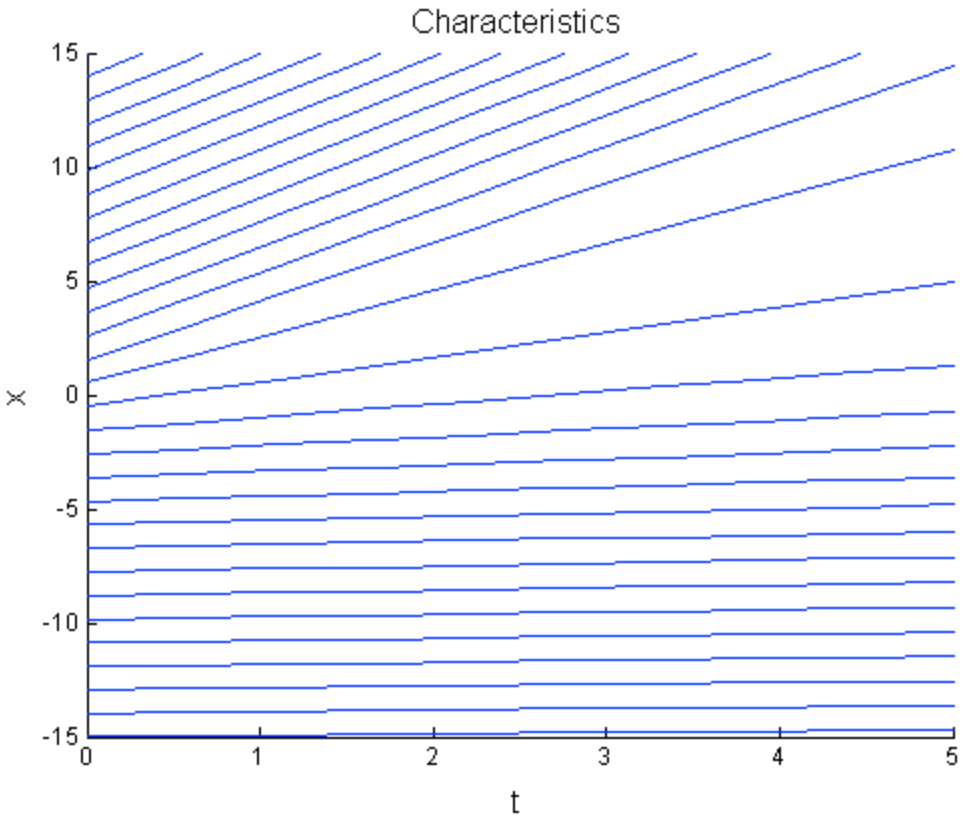
```
clear;
close all;

f = @(x) pi/2 + atan(x);

x = linspace(-15,15, 200);
t = linspace(0, 5, 10);

% Create 30 characteristic curves
xi = linspace(x(1), x(end), 30);

figure(1)
hold on;
for j = 1:length(xi)
    plot(t, f(xi(j))*t + xi(j), 'b');
end
axis([0 t(end) x(1) x(end)]);
xlabel('t', 'FontSize', 16);
ylabel('x', 'FontSize', 16);
title('Characteristics', 'FontSize', 16);
```

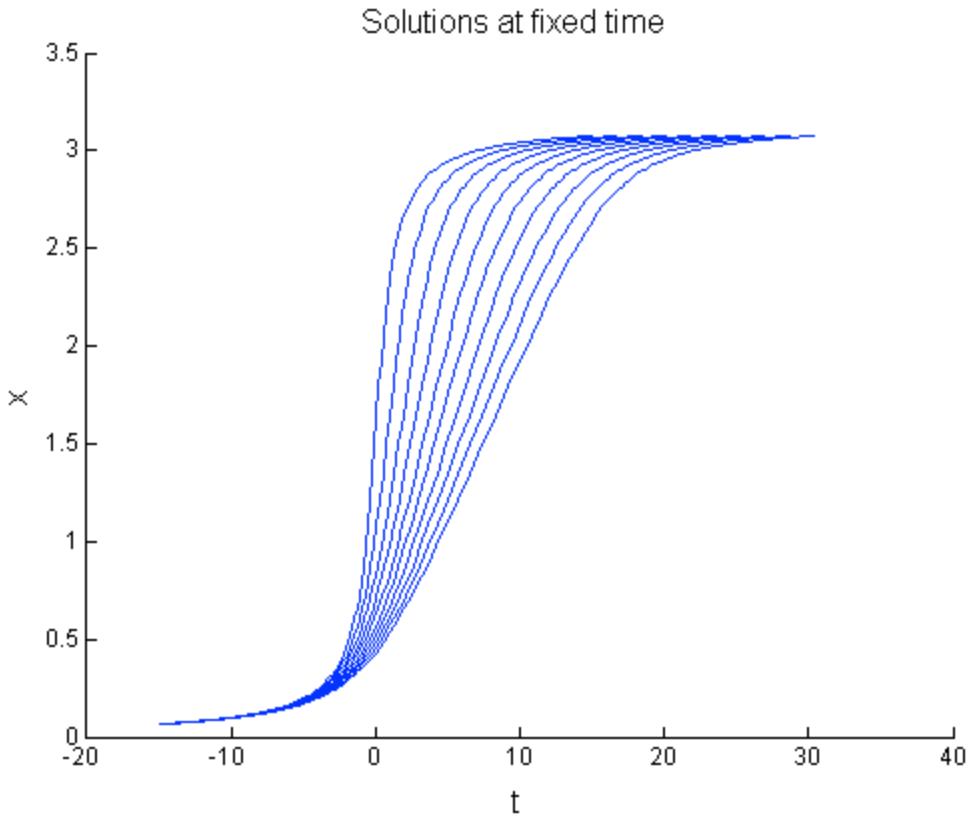


Plot snapshots of the solution

The solution is given by $u(t,x) = f(x-ut)$ and is constant along characteristic curves. If $\xi = x - ut$, and the initial condition is $u(0,x) = f(x)$, then the solution takes the value $u = f(\xi)$ along characteristics.

Here, we use $\xi = x(t=0)$ (i.e. take the original values of x to generate the characteristic curves).

```
figure(2)
hold on;
for j = 1:length(t)
    newx = x + f(x)*t(j);
    plot(newx, f(x), 'b.-');
    pause(0.2);
    drawnow
end
xlabel('t', 'FontSize', 16);
ylabel('x', 'FontSize', 16);
title('Solutions at fixed time', 'FontSize', 16);
```



Surface plotting

What does the u(t,x) surface look like? The difficulty here is that our characteristics were previously controlled by picking values of $\xi = x - f(\xi)t$. However, this produces different spacings of x depending on the current time. This is fine, but our surface will not have evenly spaced points in x

```
% Remesh things since we want enough points to be smooth
x = linspace(-15,15, 200);
t = linspace(0, 5, 100);
[X, T] = meshgrid(x,t);

% Solution matrix
U = 0*X;

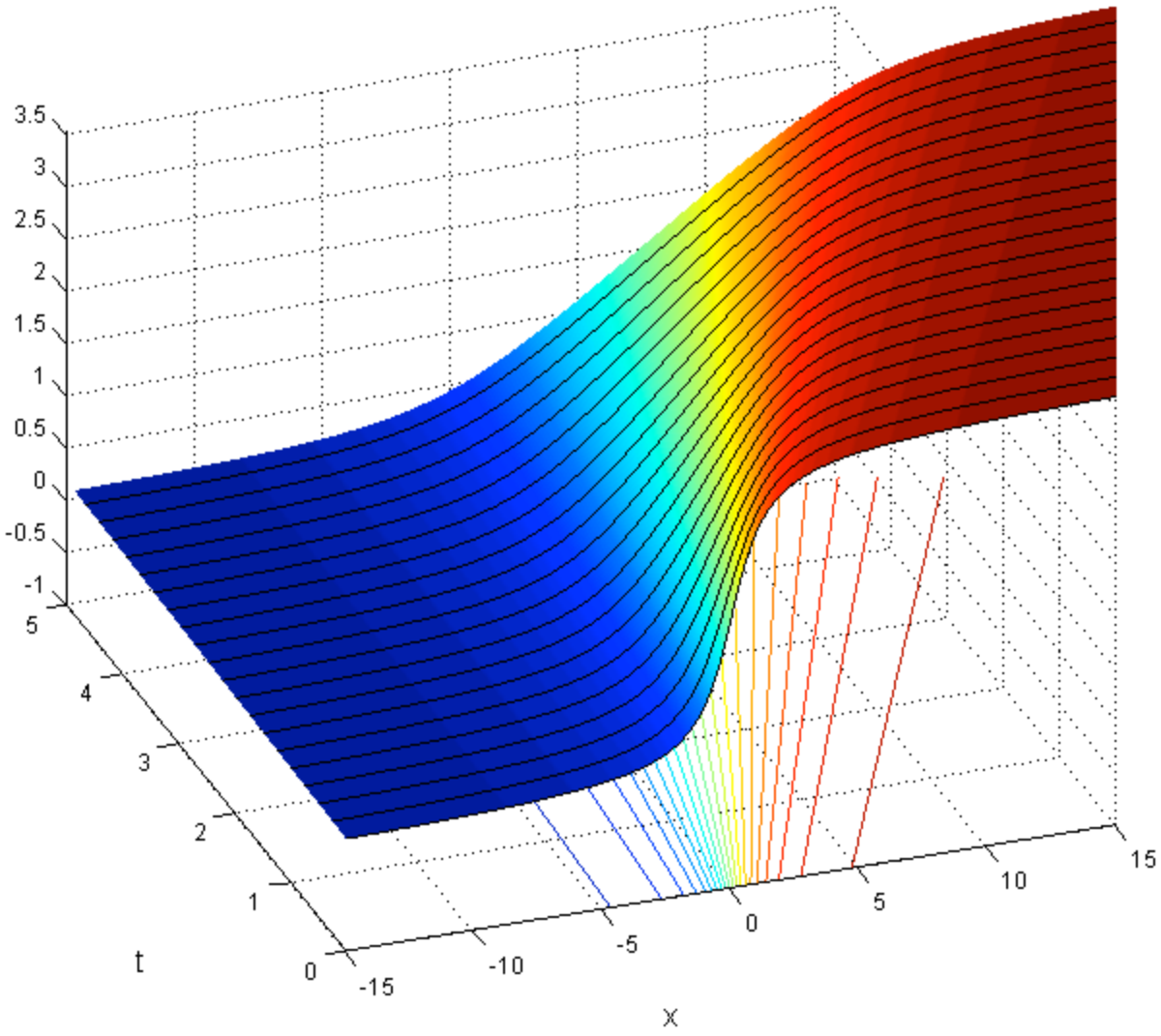
for j = 1:length(t)
    newx = x + f(x)*t(j);
    newu = f(x);

    X(j,:) = newx;  % Overwrite the points in x
    U(j,:) = newu;
end

% Everything below is from the previous Matlab sheet
figure(3)
S = surfc(X, T, U);
set(S, 'EdgeColor', 'none', 'FaceColor', 'interp');
```

```
% Outline the profiles and make the characteristics
hold on
for j = 1:5:length(t)
    plot3(X(j,:), t(j)*ones(1,length(x)), U(j,:), 'k');
end
[cc,hh] = contour3(X,T,U, 20);
zpos = -1;
for i = 1:length(hh)
        zz = get(hh(i),'Zdata');
        set(hh(i),'Zdata',zpos*ones(size(zz)));
end
hold off


% Make everything pretty
view([-20 38]);
xlim([-15 15]);
set(gcf, 'Color', 'w', 'Units', 'pixels', 'Position', [200 200 700 600]);
xlabel('x', 'FontSize', 16);
ylabel('t', 'FontSize', 16);
```



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Contents

Mini-Matlab Lesson 16: Nonlinear transport equations (Q2)

We plot characteristics and solutions for the PDE

$u_t + uu_x = 0$

with $u(0,x) = f(x) = \pi/2 - tan^{-1}(x)$.

The solution is $u(t,x) = f(\xi) = f(x - ut)$

```
clear;
close all;

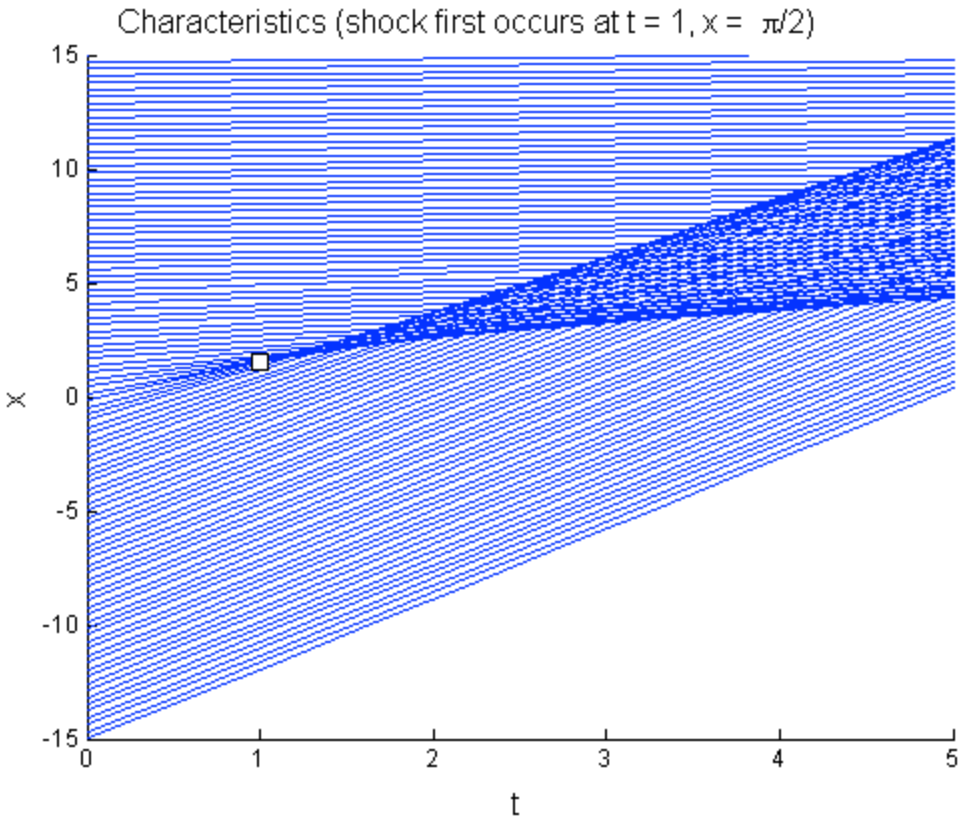
f = @(x) pi/2 - atan(x);

x = linspace(-15,15, 200);
t = linspace(0, 5, 10);

% Create 100 characteristic curves
xi = linspace(x(1), x(end), 100);

figure(1)
hold on;
for j = 1:length(xi)
    plot(t, f(xi(j))*t + xi(j), 'b');
end
plot(1, pi/2, 'ks', 'MarkerSize', 10, 'MarkerFaceColor', 'w');

axis([0 t(end) x(1) x(end)]);
xlabel('t', 'FontSize', 16);
ylabel('x', 'FontSize', 16);
title('Characteristics (shock first occurs at t = 1, x = \pi/2)', 'FontSize', 16);
```



Plot snapshots of the solution

The solution is given by $u(t,x) = f(x-ut)$ and is constant along characteristic curves. If $\xi = x - ut$, and the initial condition is $u(0,x) = f(x)$, then the solution takes the value $u = f(\xi)$ along characteristics.

Here, we use xi = original x along the t = 0 axis

```
figure(2)
hold on;
for j = 1:length(t)
    newx = x + f(x)*t(j);
    plot(newx, f(x), 'b.-');
    pause(0.2);
    drawnow
end
xlabel('t', 'FontSize', 16);
ylabel('x', 'FontSize', 16);
title('Solutions at fixed time', 'FontSize', 16);
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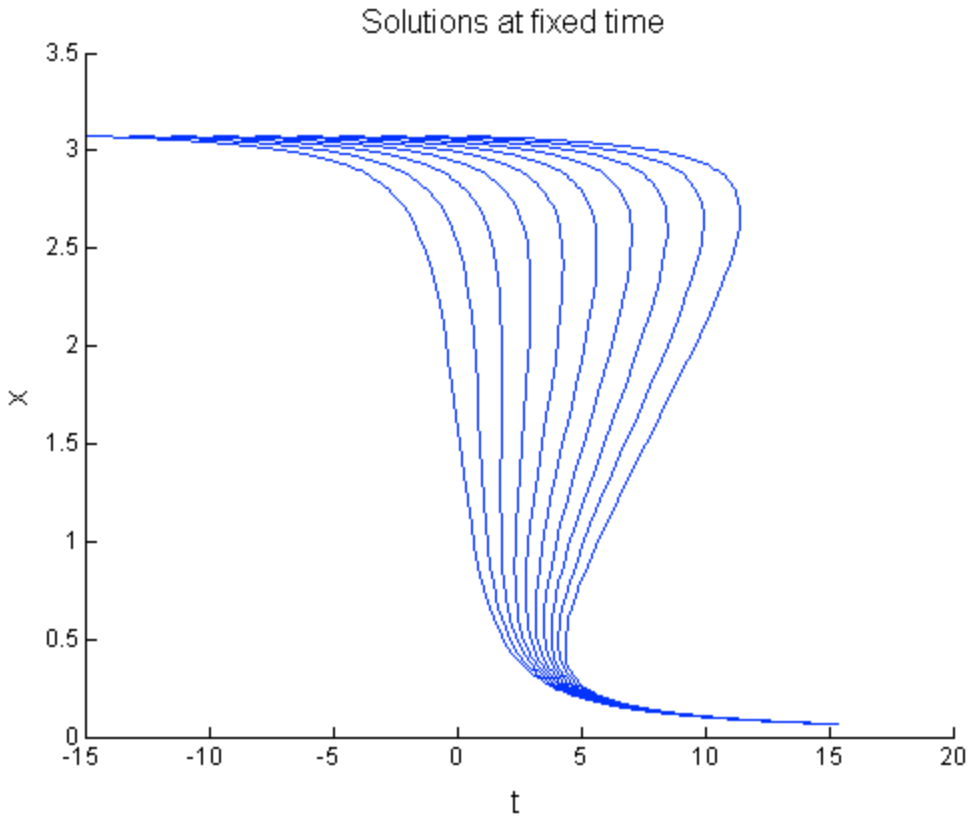


Surface plotting

```
% Remesh things since we want enough points to be smooth
x = linspace(-15,15, 200);
t = linspace(0, 5, 100);
[X, T] = meshgrid(x,t);

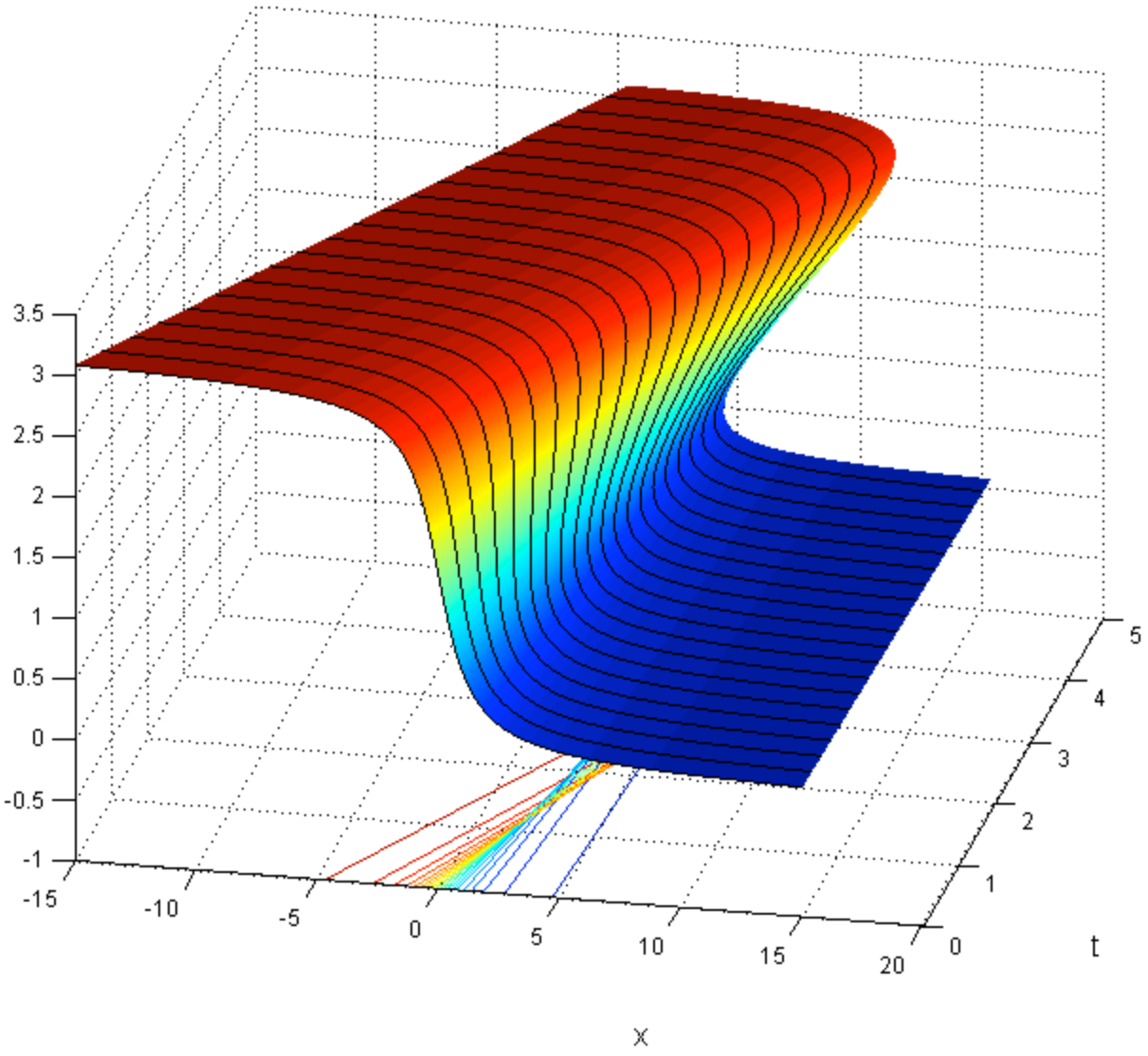
% Solution matrix
U = 0*X;
for j = 1:length(t)
    newx = x + f(x)*t(j);
    newu = f(x);
    X(j,:) = newx;
    U(j,:) = newu;
end
```

```
% Everything below is from the previous Matlab sheet
figure(3)
S = surfc(X, T, U);
set(S, 'EdgeColor', 'none', 'FaceColor', 'interp');


% Outline the profiles and make the characteristics
hold on
for j = 1:5:length(t)
    plot3(X(j,:), t(j)*ones(1,length(x)), U(j,:), 'k');
end
```

```
[cc,hh] = contour3(X,T,U, 20);
zpos = -1;
for i = 1:length(hh)
        zz = get(hh(i),'Zdata');
        set(hh(i),'Zdata',zpos*ones(size(zz)));
end
hold off


% Make everything pretty
view([12 30]);
set(gcf, 'Color', 'w', 'Units', 'pixels', 'Position', [200 200 700 600]);
xlabel('x', 'FontSize', 16);
ylabel('t', 'FontSize', 16);
```



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