

## Mini-Matlab Lesson 15a: Transport equations

This Matlab code plots the surfaces and characteristics we will discuss in the lecture for the non-uniform transport equation:

$$u_t + \left( \frac{1}{x^2 + 1} \right) u_x = 0$$

with  $u(0, x) = 1/[1 + (x + 2.75)^2]$ .

```
clear;
close all;

f = @(x) 1./(1+(x+2.75).^2);

x = linspace(-4,6, 800);
t = linspace(0, 40, 40);

[X, T] = meshgrid(x,t);
K = X.^3/3 + X - T;

U = f(K);

S = surfc(X, T, U);

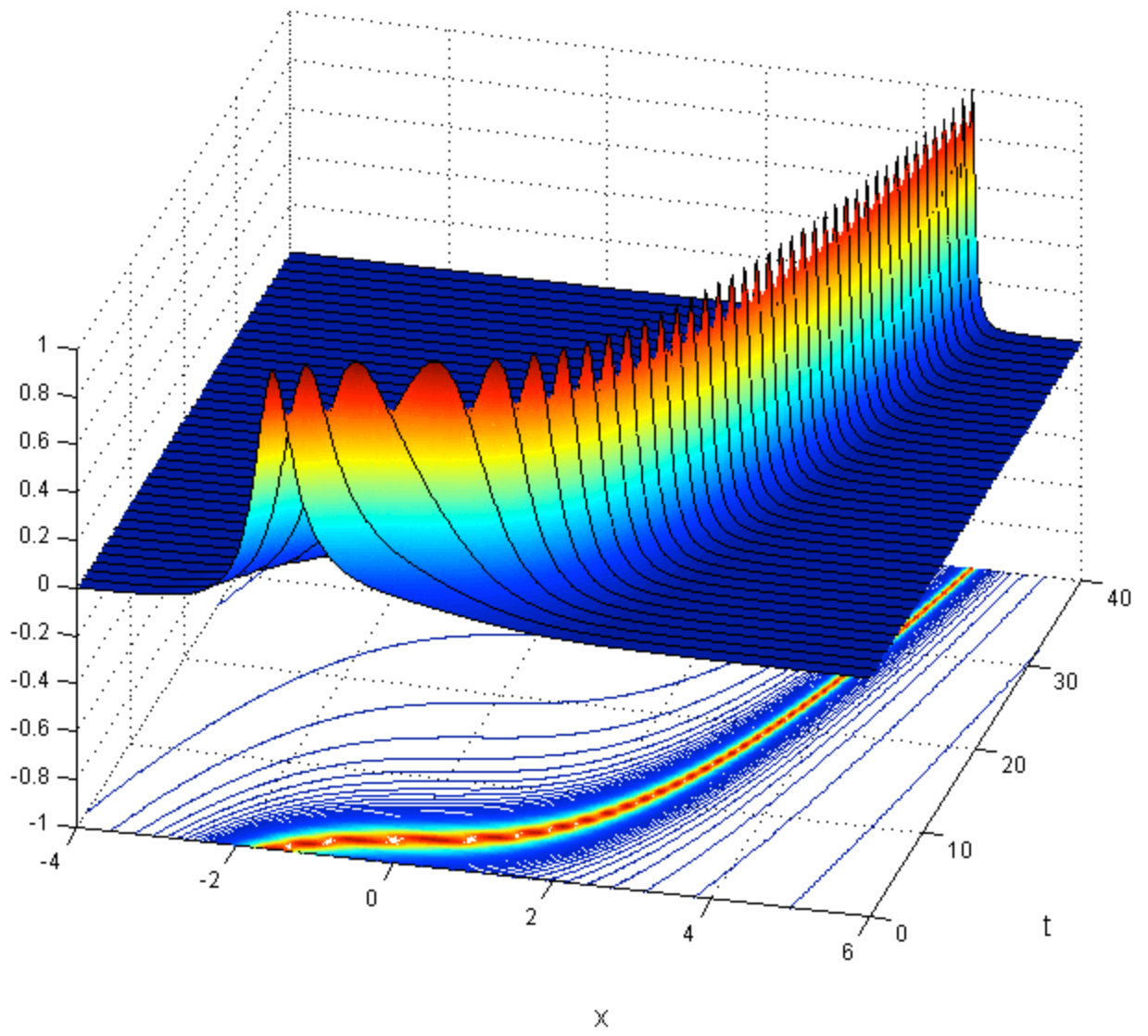
% Remove edges (too messy) and make colors nice
set(S, 'EdgeColor', 'none', 'FaceColor', 'interp');

hold on

% This creates the outlines of the profiles
for j = 1:length(t)
    plot3(x, t(j)*ones(1,length(x)), U(j,:), 'k');
end

% This complicated bit draws the characteristics. Not really necessary to
% do it this way, and an easier way is to draw the characteristics using
% the explicit formula
[cc,hh] = contour3(X,T,U, linspace(0,1,50).^2);
zpos = -1;
for i = 1:length(hh)
    zz = get(hh(i), 'Zdata');
    set(hh(i), 'Zdata', zpos*ones(size(zz)));
end
hold off

% Make everything pretty
view([15 36]);
set(gcf, 'Color', 'w', 'Units', 'pixels', 'Position', [200 200 700 600]);
xlabel('x', 'FontSize', 16);
ylabel('t', 'FontSize', 16);
```



## Mini-Matlab Lesson 15b: Transport equations

This Matlab code plots the surfaces and characteristics we will discuss in the lecture for the non-uniform transport equation:

$$u_t + (x^2 - 1)u_x = 0$$

$$\text{with } u(0, x) = e^{-x^2}.$$

```
clear;
close all;

u = @(x,t) exp(-((exp(2*t).*(x+1)+x-1)./(exp(2*t).*(x+1)-x+1)).^2);

x = linspace(-5,5, 200);
t = linspace(0, 6, 20);

[X, T] = meshgrid(x,t);
K = X.^3/3 + X - T;

U = u(X, T);

% We need to replace all the elements in the shaded region by 0
for j = 1:length(t)
    for k = 1:length(x)
        if x(k) <= (1+exp(2*t(j)))/(1-exp(2*t(j)))
            U(j,k) = 0;
        end
    end
end

S = surf(X, T, U);
set(S, 'EdgeColor', 'none', 'FaceColor', 'interp');

% Outline the profiles and make the characteristics
hold on
for j = 1:length(t)
    plot3(x, t(j)*ones(1,length(x)), U(j,:), 'k');
end
[cc,hh] = contour3(X,T,U);
zpos = -1;
for i = 1:length(hh)
    zz = get(hh(i), 'Zdata');
    set(hh(i), 'Zdata', zpos*ones(size(zz)));
end
hold off

% Make everything pretty
view([49 36]);
set(gcf, 'Color', 'w', 'Units', 'pixels', 'Position', [200 200 700 600]);
xlabel('x', 'FontSize', 16);
ylabel('t', 'FontSize', 16);
```

