1. PS1, Q6
   (a) Clearly explain how to form the equation of a plane when given a normal vector and the requirement that the plane contains a single point (x_0, y_0, z_0).
   (b) Clearly explain how to form the equation of a plane when given the requirement that the plane contains three specified points.
   (c) Find the equation for the tangent plane to the surface $2xz^2 - 3xy - 4x = 7$ at $(1, -1, 2)$.

2. PS1, Q7
   (a) A mass distribution in the x region of the xy-plane and in the shape of a semi-circle of radius a centred on the origin, has mass per unit area k. Find, using plane polar coordinates, (i) its mass, M, (ii) the coordinates of its centre of mass, (iii) its moments of inertia about the x and y axes.
   (b) Do as above for a semi-infinite sheet with mass per unit area

$$\sigma = k \exp[-(x^2 + y^2)/a^2] \quad \text{for } x \geq 0, \sigma = 0 \text{ for } x < 0$$

3. PS2, Q5

   The thermodynamic relation $\delta q = C_V \, \mathrm{d}T + (RT/V)\mathrm{d}V$ is not an exact differential. **Why?** Show that by dividing this equation by T, it becomes exact.

4. PS2 Q6

   Find the surface area of the curved portion of a hemisphere of radius a by
   (a) directly integrating the element of area $a^2 \sin\theta \mathrm{d}\theta \mathrm{d}\phi$ over the surface of the hemisphere.
   (b) projecting onto an integral taken over the xy plane.

5. PS3 Q2

   A solid hemisphere of uniform density k occupies the volume $x^2 + y^2 + z^2 \leq a^2$, $z \geq 0$. Find (i) its total mass, the position of its centre-of-mass, and its moments and products of inertia, I_{xx}, I_{yy}, I_{zz}, I_{xy}, I_{yz}, I_{zx}.

6. PS3 Q5

   Evaluate $\int \mathbf{A} \cdot \mathbf{n} \, \mathrm{d}S$ for $\mathbf{A} = [6z, 2x + y, -x]$ and S is the entire surface of the region bounded by the cylinder $x^2 + z^2 = 0$, $x = 0$, $y = 0$, $z = 0$, and $y = 8$.

7. PS3, Q6

   The vector $\mathbf{A}$ is a function of position $\mathbf{r} = (r, y, z)$ and has components (xy^2, x^2, yz). Calculate the surface integral $\int \mathbf{A} \cdot \mathrm{d}\mathbf{S}$ over each face of the triangular prism bounded by the planes $x = 0$, $y = 0$, $z = 0$, $x + y = 1$, and $z = 1$. Show that the integral $\int \mathbf{A} \cdot \mathrm{d}\mathbf{S}$ taken outwards over the whole surface is not zero. Show that it equals $\int \nabla \cdot \mathbf{A} \, \mathrm{d}V$ calculated over the volume of the prism. Why?

8. PS4, Q5 Verify Stokes' Theorem for the vector $\mathbf{A} = (y, -x, z)$ and the hemispherical surface $\mathbf{r} = 1$, $z \geq 0$.

9. PS4, Q6 Let C be any closed loop on the surface of the cylinder $(x - 3)^2 + y^2 = 2$. Find the line integral $\int_C \mathbf{A} \cdot \mathrm{d}\mathbf{r}$ for $\mathbf{A} = (y, -x, 0)$.

10. PS4, Q9 If $\phi = 2xyz^2$, $\mathbf{F} = (xy, -z, x^2)$, and C is the curve $x = t^2$, $y = 2t$, $z = t^3$ from $t = 0$ to $t = 1$, evaluate the line integrals a) $\int_C \phi \, \mathrm{d}\mathbf{r}$; b) $\int_C \mathbf{F} \times \mathrm{d}\mathbf{r}$.