In[280]:=
```
(* Mathematica code for Oxford's Fourier Series and PDES *)
(* Week 2 -- Hilary 2013 *)
Quit[];
```

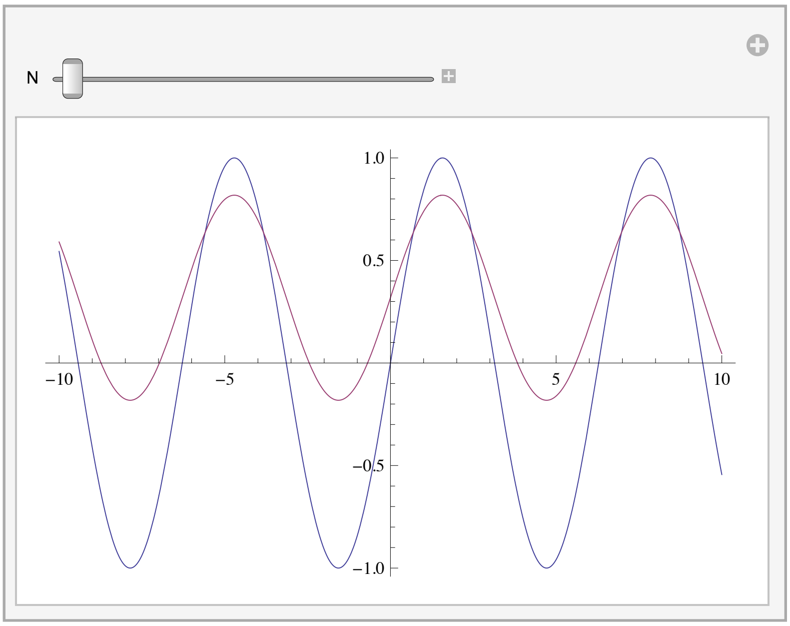
In[31]:=
```
f[x_] = Exp[x];
a0 = 1 / Pi * Integrate[f[x], {x, -Pi, Pi}];
a[k_] = FullSimplify[Integrate[f[x] * Cos[k * x], {x, -Pi, Pi}], k ∈ Integers]
b[k_] = FullSimplify[Integrate[f[x] * Sin[k * x], {x, -Pi, Pi}], k ∈ Integers]
sumf[x_, N_] := a0 / 2 + Sum[a[k] * Cos[k * x] + b[k] * Sin[k * x], {k, 1, N}]
(* Output the first three Fourier modes *)
sumf[x, 3]
Manipulate[Plot[{f[x], sumf[x, N]}, {x, -10, 10}], {N, 1, 50, 1}]
```

Out[33]= $\frac{2\ (-1)^k \text{Sinh}[\pi]}{1+k^2}$

Out[34]= $-\frac{2\ (-1)^k\ k\ \text{Sinh}[\pi]}{1+k^2}$

Out[36]= $\frac{-e^{-\pi}+e^{\pi}}{2\ \pi} - \text{Cos}[x]\ \text{Sinh}[\pi] + \frac{2}{5}\ \text{Cos}[2\ x]\ \text{Sinh}[\pi] - \frac{1}{5}\ \text{Cos}[3\ x]\ \text{Sinh}[\pi] + \text{Sin}[x]\ \text{Sinh}[\pi] - \frac{4}{5}\ \text{Sin}[2\ x]\ \text{Sinh}[\pi] + \frac{3}{5}\ \text{Sin}[3\ x]\ \text{Sinh}[\pi]$

Out[37]=



In[45]:=
```
(* For piecewised defined functions, Mathematica is a bit touchy...*)
f[x_] = Sin[x];
a0 = 1 / Pi * Integrate[f[x], {x, 0, Pi}];
a[k_] = FullSimplify[1 / Pi * Integrate[f[x] * Cos[k * x], {x, 0, Pi}]]
b[k_] = FullSimplify[1 / Pi * Integrate[f[x] * Sin[k * x], {x, 0, Pi}]]
sumf[x_, N_] := a0 / 2 +
  Sum[Limit[a[s], s → k] * Cos[k * x] + Limit[b[s], s → k] * Sin[k * x], {k, 1, N}]
sumf[x, 3]
Plot[Evaluate[Table[sumf[x, k], {k, 1, 5, 1}]], {x, -10, 10}]
```

Out[47]= $\frac{1 + \text{Cos}[k\,\pi]}{\pi - k^2\,\pi}$

Out[48]= $\frac{\text{Sin}[k\,\pi]}{\pi - k^2\,\pi}$

Out[50]= $\frac{1}{\pi} - \frac{2\,\text{Cos}[2\,x]}{3\,\pi} + \frac{\text{Sin}[x]}{2}$

Out[51]=