STOKES' PARADOX

In the slow flow approximations:

$$\underbrace{\epsilon(\underline{u}\cdot\nabla)\underline{u}}_{\text{we ignore inertial}} = \underbrace{-\nabla p + \nabla^2 \underline{u}}_{\text{we balance pressure \& viscosity.}}$$

But a typical inertial term is like: $\epsilon u u_x$ (for example); so if x is LARGE, this term may still be $O(1)$, and thus balance the viscosity.

The Stokes Paradox (which arises at leading order for a circular cylinder and second order for a sphere) occurs because Stokes' slow flow approximation <u>breaks down</u> as $r \to \infty$.

Oseen realised that we should <u>keep</u> the inertial terms (as they will affect the matching as $r \to \infty$). If we let

$$u = U + u' \qquad v = v' \qquad w = w'$$

The NS eqns have

$$\epsilon \cdot U \frac{\partial u'}{\partial x} + \ldots = -\frac{\partial p}{\partial x} + \nabla^2 u \quad \left(\begin{array}{l}x\text{-momentum,}\\ \text{for example}\end{array}\right)$$

Now we solve these OSEEN'S EQUATIONS with $u' = v' = w' = 0$ at ∞ and $u' = -U$, $v' = w' = 0$ on the sphere. The key is that terms of $O\left(\frac{1}{\log(\frac{1}{\epsilon})}\right)$ appear, which are necessary in order to match properly.